

Lecture notes on risk management, public policy, and the financial system

How asset prices express the risk of extreme events

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Risk and expectations in asset prices

Market-based measures of market risk

Market-based measures of credit and liquidity risk

Funding liquidity indicators

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The information in asset prices

Risk and expectations in asset prices

Implied volatility and risk

Market-based measures of market risk

Market-based measures of credit and liquidity risk

Funding liquidity indicators

What information are we looking for?

- In efficient markets, asset prices impound information and points of view widely dispersed among market participants
- Asset prices inherently forward-looking
- Asset prices summarize not only dispersed information about current and future conditions, but also about future asset prices themselves
- Germane to risk measurement because may give us an external, market-adjusted assessment of future to compare with model-generated
- Chief challenge to extracting information about future asset prices from current asset prices: asset prices also heavily affected by risk

Information and asset types

Cash securities and exchange rates embed information about future price of the asset and on size and certainty of future cash flows

Interest rates contain information about future interest rates as well as current and future business conditions

Futures and forwards contain information about future prices

Options contain information about probability distributions of future prices

Information in prices of futures and forwards

- Futures and forwards have future maturity dates, appear to provide distinct information point regarding future underlying price
- For long a focus of efficient markets research
 - Basic conclusion: add little information about future prices to what we know from current prices
 - Equal to current price adjusted for carry, so little to add
- Interest rates contain forwards on future short-term rates

Risk aversion and asset prices

- Asset prices express “consensus” risk preferences
 - If market participants seek insurance, assets with high payoffs in bad times are dear
 - And even more so if market participants risk averse
- Expectations/views on probability distribution of future outcomes entangled with risk preferences
- But overall strong evidence of risk aversion
- Option prices most revealing because state-dependent

Risk premiums

- Asset prices embed **risk premiums**
- Risk premiums express difference between expected values of risky and risk-free securities
 - Expected returns on risky asset higher than risk-free rate
 - \Leftrightarrow Risky assets cheaper than safe ones with same average cash flows, since income discounted by risk-free rate *plus* positive risk premium
- Risk premiums not directly observable, must be estimated
- CAPM:
 - Simple explanation of risk premium as compensation for holding market portfolio, an asset with low payoffs in bad times
 - And simple way to measure, via beta to market portfolio

Risk-neutral and subjective distributions

Subjective distribution: also called **physical** or actuarial or simply “real-life” distribution

- It is either the distribution “the market” believes in or the actual distribution from the point of view of superior knowledge

Risk-neutral distribution is the distribution implied by asset prices

- Odd name, since expresses risk aversion
- Subjective distribution we would attribute to representative agent if she is assumed indifferent to risk

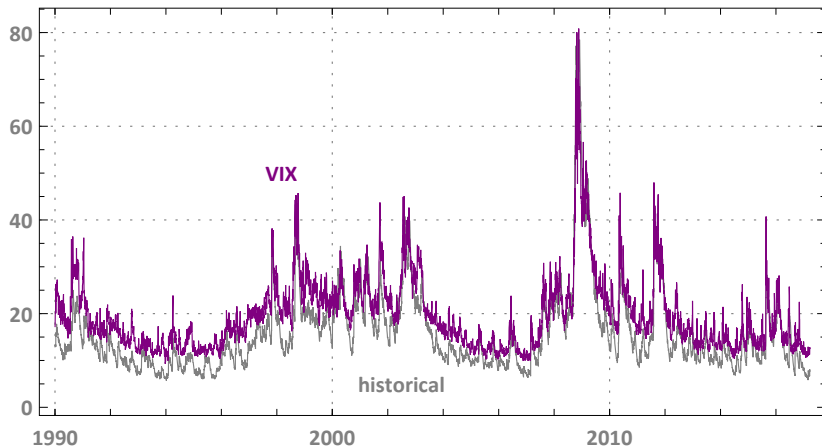
Option prices and implied volatility

- Options have asymmetric payoff function \Rightarrow option value a function of volatility
 - \Rightarrow Option prices embed a volatility forecast
- **Black-Scholes formula**: benchmark for option valuation
- **Implied volatility** := volatility estimate that matches an observed option price to the Black-Scholes formula
- Implied volatility used by option traders as a pricing metric as well as a volatility estimate

Implied volatility and risk premiums

- General *level* of implied volatility contains a negative **variance risk premium**
- Options have high payoffs in bad times, hence dearer than “fair bet”
- Implied volatility consistently higher than expected (as measured by historical)
- Analogy to insurance: probability of fire 1 in 1,000,000, but priced as if 1 in 500,000

Variance risk premium



Black plot, estimated volatility (EWMA with decay factor $\lambda = 0.94$). Red plot: VIX. Both in percent at an annual rate.

Implied volatility surface

- **Black-Scholes model**—as opposed to the *formula*—incorporates standard asset return model
 - Envisions a single unvarying implied volatility
 - Across time, option tenors, strike prices
- But standard model doesn't hold precisely
- Variation over time
- → **Implied volatility surface**: variation of implied volatility by *option maturity* and by *exercise price*

Term structure of implied volatility: long-dated options generally closer to “forever volatility”

Implied volatility skew Out-of-the-money call option volatilities \neq those of equally out-of-the-money puts

Implied volatility smile Out-of-the-money options volatilities $>$ those of at-the-money options

Options and the risk-neutral distribution

- Options state-dependent, so express risk aversion to specific outcomes
- **Empirical pricing kernel** “distributes” risk premium across price axis
 - Defined as ratio of subjective to risk neutral probability of a specific range of outcomes
- Leads to a forecast of entire return distribution (**Breeden-Litzenberger formula**)
- Estimate of risk-neutral distribution
- → Alternatives to the standard return model

Breeden-Litzenberger formula

- Option prices embed a risk-neutral distribution (RND)
- $c(t, X, \tau) \equiv$ implied volatility surface, the market's time- t price schedule of European calls with tenor τ , struck at X
- $\tilde{\Pi}(X) \equiv$ time- t risk-neutral CDF, looking τ years ahead

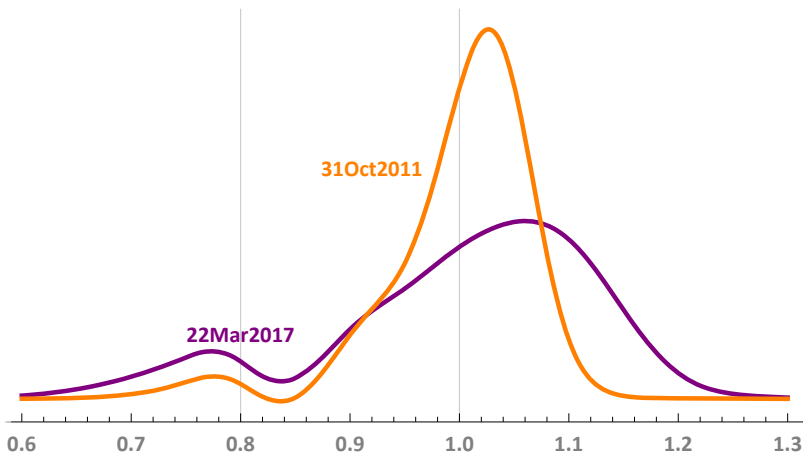
$$\tilde{\Pi}(X) = 1 + e^{r\tau} \frac{\partial c}{\partial X}$$

- Intuitively, CDF \approx 1 minus the slope of the call value as a function of strike
- Equivalently via Black-Scholes implied volatility smile $\sigma(t, X, \tau)$ and relationship to Black-Scholes formula $c(t, X, \tau) = v[X, \tau, \sigma(t, X, \tau), \dots]$

Challenges arise from limited data

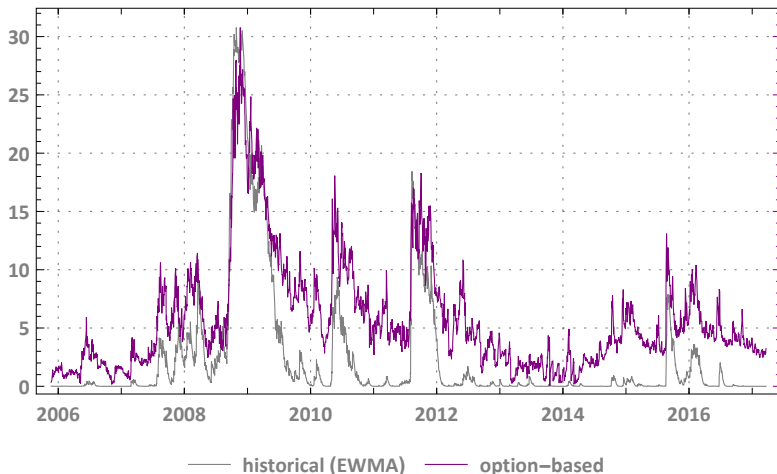
- In principle not hard to compute as discrete approximation
- Ideal data requirements to construct RND for specific asset/horizon
 - Dense set of plain-vanilla European options with same tenor but varying strikes, covering support of distribution
 - Ross theorem may permit using less data
- Real-world limitations: paucity and poor quality of data
 - Gaps in both strike and maturity dimensions
 - Sometimes no data, but even when available, need to fit or interpolate
 - Tails generally poorly represented or not at all
 - Liquidity: at any time, trading focuses on just a few strikes
 - Data problems → violations of no-arbitrage conditions

Risk-neutral distribution of S&P 500 index



Density function of arithmetic return of the S&P 500 index 3 months hence. Black plot, using the estimated volatility (EWMA with decay factor $\lambda = 0.97$) on Oct. 31, 2011. Red (blue) plot: risk-neutral, estimated using the 3-month implied volatility smiles on Oct. 31, 2011 (Aug. 29, 2012).

Risk-neutral and subjective tail risk



Probability of a decline of at least 20 percent in the S&P 500 index over the subsequent 3 months. Black plot, based on estimated volatility (EWMA, decay factor $\lambda = 0.94$). Red plot: risk-neutral, estimated using the 3-month implied volatility smile.

Correlation in market prices

Equity prices: correlation between returns on individual stocks

Interest rates: correlation between rates for different tenors \Leftrightarrow likelihood of changes in term spread

Credit spreads: default correlation

Definition and asset class coverage

- Correlations implied by derivatives prices
- Option-implied correlations
 - Equity markets: return correlation implied by index and single-stock vols
 - Analogous construction for FX: infer correlations of 2 USD currency pairs from cross-currency options
 - Fixed-income markets: correlation of changes in yields of two swap maturities, implied by swaption and curve option prices
- Implied default correlations
 - Correlation of default for two different firms
 - Implied by prices of two standard tranches

Interpretation and limitations

- Related to market assessment of relative importance of idiosyncratic and systematic risk
- Index vols high relative to typical single-stock vol \Rightarrow market participants (risk-neutrally) eager to cover generic equity exposure \rightarrow high implied correlation
 - For any set of firms, expectation of large common shocks \Rightarrow higher correlation
 - Generally, high implied equity correlation in crises
- Data limitations
 - Some “constant pairwise correlation” coverage, from options on indexes
 - Very little on pairwise return and default correlation exposures

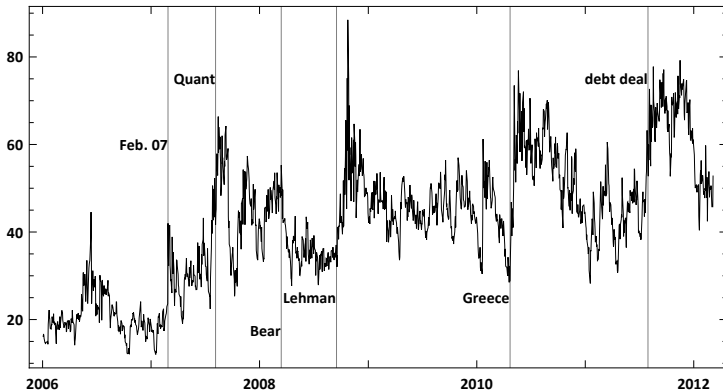
Construction of equity option-implied correlation

- Definition of stock portfolio volatility (k constituents):

$$\sigma_{\text{index},t}^2 = \sum_k \omega_{kt}^2 \sigma_{kt}^2 + 2\rho_t \sum_k \sum_{j<k} \omega_{kt} \omega_{jt} \sigma_{kt} \sigma_{jt}$$

- Construction of implied correlation
 - Substitute observed index and single-firm implied vols $\sigma_{\text{index},t}$ and σ_{kt}^2 , market cap weights ω_{kt}
 - Solve for ρ_t , constant (in cross-section) pairwise correlation

S&P 500 option-implied correlation

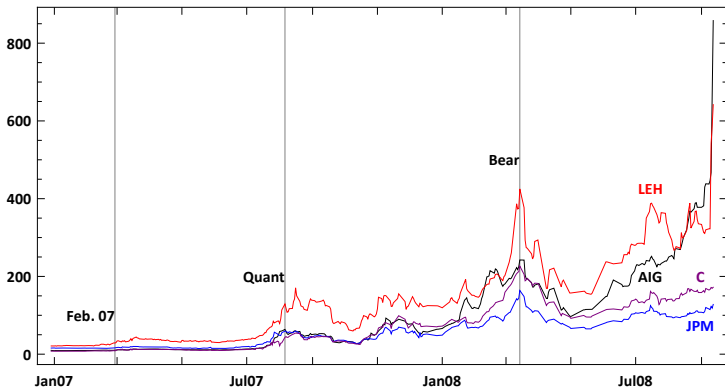


In percent, daily, 03Jan2006 to 31Oct2012. Uses the 100 largest-cap constituents (as of end.-Oct 2012) that have been in the S&P 500 index since at least Jan. 3, 2006, accounting for about 72 percent of the the total market cap of the index. *Data source:* Bloomberg Financial L.P.

Definition and interpretation

- Equivalent to spread of bond yield over swap/Treasury curve
 - But CDS floating-payment trigger may not be identical to a bond's default event
 - Similar to forwards in being issued daily with uniform tenor rather than fixed maturity date
- Indicator of risk-neutral default expectations
 - CDS curve can be used to extract time-varying hazard rates
 - But requires assumptions/estimates of recovery rate
- Spread to congruent bond yield an indicator of funding liquidity conditions
 - Exposure can be maintained with leverage
- Comparison to equity tail risk can be revealing

Financial CDS spreads

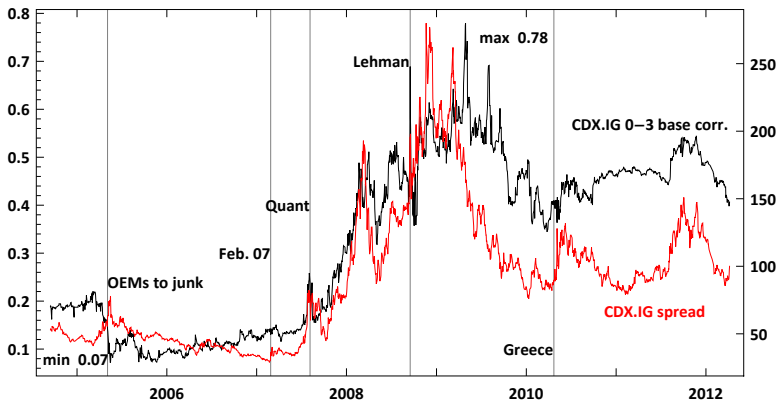


American International Group, Lehman Brothers Holdings Inc., JPMorgan Chase & Co. and Citigroup Inc. Senior unsecured, 5-year tenor, in bps. Daily end-2006 to 12Sep08. Source: Bloomberg Financial L.P.

Definition and interpretation

- Underlying an equally-weighted basket of CDS, e.g. CDX.NA.IG or iTraxx
 - Spread an indicator of average/overall risk-neutral default expectations
- Standard tranches
 - Exposure to segments of loss: 0–3 percent, 3–7 percent, etc.
 - **Base correlation**: copula correlation that matches model to market price
 - May be interpreted as an “index” of risk-neutral default correlation
 - **Correlation skew**: base correlation varies with tranche
 - Hedge ratios: notional amount of index that hedges tranche vs. small change in index spread
 - Level and volatility related to convexity in leveraged credit trades
 - Indicator of credit-market stress (2005 downgrade of Ford, GM)
 - Indicator of large positions (early 2012)

Base correlation 2004–2012



Black line (left axis) plots the equity (0-3 percent) base correlation of the on-the-run 5-year CDX.NA.IG series. Purple line (right axis) plots the on-the-run 5-year CDX.NA.IG series spread. Daily, 15Sep2004 to 31Oct2012. Source: JPMorgan.

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Money markets

Overview

- Spread between a derivatives-based and a spot rate. Examples:
 - LOIS
 - Swap to Treasury
 - Foreign-exchange forward-implied to Libor